Syllabus content

Topic 1—Algebra 8 hrs

Aims

The aim of this section is to introduce students to some basic algebraic concepts and applications. Number systems are now in the presumed knowledge section.

Details

	Content	Amplifications/inclusions	Exclusions
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation.	Examples of applications, compound interest and population growth.	
1.2	Exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	Elementary treatment only is required. Examples: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5\log 2$; $(2^3)^{-4} = 2^{-12}$. $\log_b a = \frac{\log_c a}{\log_c b}$.	
1.3	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$.	On examination papers: students may determine the binomial coefficients, $\binom{n}{r}$, by using Pascal's triangle, or by using a GDC.	The formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ and consideration of combinations.

Topic 2—Functions and equations 24 hrs

Aims

The aims of this section are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of a GDC in both the development and the application of this topic.

Details

	Content	Amplifications/inclusions	Exclusions
2.1	Concept of function $f: x \mapsto f(x)$: domain, range; image (value).	On examination papers: if the domain is the set of real numbers then the statement "$x \in \mathbb{R}$" will be omitted.	Formal definition of a function; the terms "one-to-one", "many-to-one" and "codomain".
	Composite functions $f \circ g$; identity function.	The composite function $(f \circ g)(x)$ is defined as $f(g(x))$.	
	Inverse function f^{-1}.	On examination papers: if an inverse function is to be found, the given function will be defined with a domain that ensures it is one-to-one.	Domain restriction.
2.2	The graph of a function; its equation $y = f(x)$.	On examination papers: questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus. The linear function $ax + b$ is now in the presumed knowledge section.	
	Function graphing skills:		
	use of a GDC to graph a variety of functions;		
	investigation of key features of graphs.	Identification of horizontal and vertical asymptotes.	
	Solution of equations graphically.	May be referred to as roots of equations, or zeros of functions.	

Topic 2—Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.3	Transformations of graphs: translations; stretches; reflections in the axes. The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.	Translations: $y = f(x) + b$; $y = f(x - a)$. Stretches: $y = pf(x)$; $y = f\left(x/q\right)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Examples: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the *y*-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the *y*-direction and a stretch of scale factor $\frac{1}{2}$ in the *x*-direction.	
2.4	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph; its self-inverse nature.		

Topic 2—Functions and equations (continued)

	Content	Amplifications/inclusions	Exclusions
2.5	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry $x = -\frac{b}{2a}$. The form $x \mapsto a(x-h)^2 + k$: vertex (h, k). The form $x \mapsto a(x-p)(x-q)$: x-intercepts $(p, 0)$ and $(q, 0)$.	Rational coefficients only.	
2.6	The solution of $ax^2 + bx + c = 0$, $a \neq 0$. The quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$.		On examination papers: questions demanding elaborate factorization techniques will not be set.
2.7	The function: $x \mapsto a^x$, $a > 0$. The inverse function $x \mapsto \log_a x$, $x > 0$. Graphs of $y = a^x$ and $y = \log_a x$. Solution of $a^x = b$ using logarithms.	$\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.	
2.8	The exponential function $x \mapsto e^x$. The logarithmic function $x \mapsto \ln x$, $x > 0$.	$a^x = e^{x \ln a}$. Examples of applications: compound interest, growth and decay.	

Topic 3—Circular functions and trigonometry 16 hrs

Aims

The aims of this section are to explore the circular functions and to solve triangles using trigonometry.

Details

	Content	Amplifications/inclusions	Exclusions
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as multiples of π, or decimals.	
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. The identity $\cos^2\theta + \sin^2\theta = 1$.	Given $\sin\theta$, finding possible values of $\cos\theta$ without finding θ. Lines through the origin can be expressed as $y = x\tan\theta$, with gradient $\tan\theta$.	The reciprocal trigonometric functions $\sec\theta$, $\csc\theta$ and $\cot\theta$.
3.3	Double angle formulae: $\sin 2\theta = 2\sin\theta\cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$.	Double angle formulae can be established by simple geometrical diagrams and/or by use of a GDC.	Compound angle formulae.
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; their periodic nature; and their graphs. Composite functions of the form $f(x) = a\sin(b(x+c)) + d$.	On examination papers: radian measure should be assumed unless otherwise indicated by, for example, $x \mapsto \sin x°$. Example: $f(x) = 2\cos(3(x-4)) + 1$. Examples of applications: height of tide, Ferris wheel.	The inverse trigonometric functions: $\arcsin x$, $\arccos x$ and $\arctan x$.

Topic 3—Circular functions and trigonometry (continued)

	Content	Amplifications/inclusions	Exclusions
3.5	Solution of trigonometric equations in a finite interval. Equations of the type $a\sin(b(x+c)) = k$. Equations leading to quadratic equations in, for example, $\sin x$. Graphical interpretation of the above.	Examples: $2\sin x = 3\cos x$, $0 \le x \le 2\pi$. $2\sin 2x = 3\cos x$, $0^\circ \le x \le 180^\circ$. $2\sin x = \cos 2x$, $-\pi \le x \le \pi$. Both analytical and graphical methods required.	The general solution of trigonometric equations.
3.6	Solution of triangles. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Area of a triangle as $\frac{1}{2}ab\sin C$.	Appreciation of Pythagoras' theorem as a special case of the cosine rule. The ambiguous case of the sine rule. Applications to problems in real-life situations, such as navigation.	

Topic 4—Matrices 10 hrs

Aims

The aim of this section is to provide an elementary introduction to matrices, a fundamental concept of linear algebra.

Details

	Content	Amplifications/inclusions	Exclusions
4.1	Definition of a matrix: the terms “element”, “row”, “column” and “order”.	Use of matrices to store data.	Use of matrices to represent transformations.
4.2	Algebra of matrices: equality; addition; subtraction; multiplication by a scalar. Multiplication of matrices. Identity and zero matrices.	Matrix operations to handle or process information.	
4.3	Determinant of a square matrix. Calculation of 2×2 and 3×3 determinants. Inverse of a 2×2 matrix. Conditions for the existence of the inverse of a matrix.	Elementary treatment only. Obtaining the inverse of a 3×3 matrix using a GDC.	 Cofactors and minors. Other methods for finding the inverse of a 3×3 matrix.
4.4	Solution of systems of linear equations using inverse matrices (a maximum of three equations in three unknowns).	Only systems with a unique solution need be considered.	

Topic 5—Vectors 16 hrs

Aims

The aim of this section is to provide an elementary introduction to vectors. This includes both algebraic and geometric approaches.

Details

	Content	Amplifications/inclusions	Exclusions
5.1	Vectors as displacements in the plane and in three dimensions.	Distance between points in three dimensions.	
	Components of a vector; column representation. $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following topics:		
	the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$.	
	multiplication by a scalar, $k\boldsymbol{v}$;		
	magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	unit vectors; base vectors $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$;		
	position vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$.	$\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$.	

Topic 5—Vectors (continued)

	Content	Amplifications/inclusions	Exclusions
5.2	The scalar product of two vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$; $\boldsymbol{v}\cdot\boldsymbol{w}=v_1w_1+v_2w_2+v_3w_3$.	The scalar product is also known as the “dot product” or “inner product”.	Projections.
	Perpendicular vectors; parallel vectors.	For non-zero perpendicular vectors $\boldsymbol{v}\cdot\boldsymbol{w}=0$; for non-zero parallel vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\pm\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.	
	The angle between two vectors.		
5.3	Representation of a line as $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$.	Lines in the plane and in three-dimensional space. Examples of applications: interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert\boldsymbol{b}\rvert$ representing speed.	Cartesian form of the equation of a line: $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$.
	The angle between two lines.		
5.4	Distinguishing between coincident and parallel lines.		
	Finding points where lines intersect.	Awareness that non-parallel lines may not intersect.	

Topic 6—Statistics and probability 30 hrs

Aims

The aim of this section is to introduce basic concepts. It may be considered as three parts: descriptive statistics (6.1–6.4), basic probability (6.5–6.8), and modelling data (6.9–6.11). It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained.

Details

	Content	Amplifications/inclusions	Exclusions
6.1	Concepts of population, sample, random sample and frequency distribution of discrete and continuous data.	Elementary treatment only.	
6.2	Presentation of data: frequency tables and diagrams, box and whisker plots. Grouped data: mid-interval values, interval width, upper and lower interval boundaries, frequency histograms.	Treatment of both continuous and discrete data. A frequency histogram uses equal class intervals.	 Histograms based on unequal class intervals.

	Content	Amplifications/inclusions	Exclusions
6.3	Mean, median, mode; quartiles, percentiles.	Awareness that the population mean, μ, is generally unknown, and that the sample mean, $\bar{x}$, serves as an estimate of this quantity.	Estimation of the mode from a histogram.
	Range; interquartile range; variance; standard deviation.	Awareness of the concept of dispersion and an understanding of the significance of the numerical value of the standard deviation.	
		Obtaining the standard deviation (and indirectly the variance) from a GDC is expected.	Other methods for finding the standard deviation or variance.
		Awareness that the population standard deviation, σ, is generally unknown, and that the standard deviation of the sample, s_n, serves as an estimate of this quantity.	Discussion of bias of s_n^2 as an estimate of σ^2.
6.4	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.		
6.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.		
	The probability of an event A as $\mathrm{P}(A) = \frac{n(A)}{n(U)}$.		
	The complementary events A and A' (not A); $\mathrm{P}(A) + \mathrm{P}(A') = 1$.		

	Content	Amplifications/inclusions	Exclusions
6.6	Combined events, the formula: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. $\mathrm{P}(A \cap B) = 0$ for mutually exclusive events.	Appreciation of the non-exclusivity of "or". Use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ for mutually exclusive events.	
6.7	Conditional probability; the definition $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$. Independent events; the definition $\mathrm{P}(A \mid B) = \mathrm{P}(A) = \mathrm{P}(A \mid B')$.	The term "independent" is equivalent to "statistically independent". Use of $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ for independent events.	
6.8	Use of Venn diagrams, tree diagrams and tables of outcomes to solve problems.		
6.9	Concept of discrete random variables and their probability distributions. Expected value (mean), $\mathrm{E}(X)$ for discrete data.	Simple examples only, such as: $\mathrm{P}(X = x) = \frac{1}{18}(4 + x)$ for $x \in \{1, 2, 3\}$; $\mathrm{P}(X = x) = \frac{5}{18}, \frac{6}{18}, \frac{7}{18}$. Knowledge and use of the formula $\mathrm{E}(X) = \sum \left(x\mathrm{P}(X = x)\right)$. Applications of expectation, for example, games of chance.	Formal treatment of random variables and probability density functions; formal treatment of cumulative frequency distributions and their formulae.

	Content	Amplifications/inclusions	Exclusions
6.10	Binomial distribution.		The formula $\binom{n}{r}=\frac{n!}{r!(n-r)!}$ and consideration of combinations.
	Mean of the binomial distribution.		Formal proof of mean.
6.11	Normal distribution.		Normal approximation to the binomial distribution.
	Properties of the normal distribution.	Appreciation that the standardized value (z) gives the number of standard deviations from the mean.	
	Standardization of normal variables.	Use of calculator (or tables) to find normal probabilities; the reverse process.	

Topic 7—Calculus 36 hrs

Aims

The aim of this section is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

Details

	Content	Amplifications/inclusions	Exclusions
7.1	Informal ideas of limit and convergence.	Only an informal treatment of limit and convergence, for example, 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$.	
	Definition of derivative as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of polynomial functions only. Other derivatives can be justified by graphical considerations using a GDC. Familiarity with both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative.	
	Derivative of x^n $(n\in\mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Derivative interpreted as gradient function and as rate of change.	Finding equations of tangents and normals. Identifying increasing and decreasing functions.	

Topic 7—Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.2	Differentiation of a sum and a real multiple of the functions in 7.1. The chain rule for composite functions. The product and quotient rules. The second derivative.	Familiarity with both forms of notation, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ and $f''(x)$, for the second derivative.	
7.3	Local maximum and minimum points. Use of the first and second derivative in optimization problems.	Testing for maximum or minimum using change of sign of the first derivative and using sign of the second derivative. Examples of applications: profit, area, volume.	
7.4	Indefinite integration as anti-differentiation. Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x. The composites of any of these with the linear function $ax+b$.	$\int \frac{1}{x}\mathrm{d}x = \ln x + C$, $x > 0$. Example: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	

Topic 7—Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.5	Anti-differentiation with a boundary condition to determine the constant term.	Example: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	
	Definite integrals.		
	Areas under curves (between the curve and the x-axis), areas between curves.	Only the form $\int_a^b y\,dx$.	$\int_a^b x\,dy$.
	Volumes of revolution.	Revolution about the x-axis only, $V = \int_a^b \pi y^2\,dx$.	Revolution about the y-axis; $V = \int_a^b \pi x^2\,dy$.
7.6	Kinematic problems involving displacement, s, velocity, v, and acceleration, a.	$v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. Area under velocity–time graph represents distance.	
7.7	Graphical behaviour of functions: tangents and normals, behaviour for large $\lvert x\rvert$,	Both “global” and “local” behaviour.	
	horizontal and vertical asymptotes.		Oblique asymptotes.
	The significance of the second derivative; distinction between maximum and minimum points.	Use of the terms “concave-up” for $f''(x) > 0$, “concave-down” for $f''(x) < 0$.	
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion $f''(x) = 0$ and $f''(x)$ changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	Points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.